B. Sc. (Hons.) Part I Paper 2nd

Alok kr
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Integral calculus

Some important (but difficult) problems

1. $$I = \int \sqrt{x^2+1}\left[\frac{\log(x^2+1) - 2\log x}{x^4}\right] dx$$

$$\Rightarrow I = \int \frac{\sqrt{x^2+1}}{x^4} \cdot \log \frac{x^2+1}{x^2}\, dx$$

$$= \int \frac{1}{x^{\cancel{3}}} \cdot \sqrt{\frac{x^2+1}{x^{\cancel{2}}}} \cdot \log\left(1+\frac{1}{x^2}\right) dx$$

$$\Rightarrow I = \int \frac{1}{x^3} \cdot \sqrt{1+\frac{1}{x^2}}\ \log\left(1+\frac{1}{x^2}\right) dx$$

Put $1+\frac{1}{x^2} = t^2$

$$\Rightarrow -\frac{\cancel{2}}{x^3}\, dx = \cancel{2}t\, dt$$

$$\Rightarrow \frac{dx}{x^3} = -t\, dt$$

$$\Rightarrow I = \int \sqrt{t^2}\ \log(t^2) \cdot (-)t\, dt$$

$$= -2\int t^2 \log t\, dt$$

$$\Rightarrow I = -2 \cdot \left[\log t \cdot \int t^2 dt - \int\left[\frac{d}{dt}(\log t) \int t^2 dt\right] dt\right]$$

$$= -2\left[\frac{t^3}{3}\log t - \int \frac{1}{t} \cdot \frac{t^3}{3}\, dt\right]$$

$$\Rightarrow I = -2\left[\frac{t^3}{3}\log t - \frac{t^3}{9}\right] + K$$

$$\Rightarrow I = -\frac{2}{3}t^3\log t + \frac{2}{9}t^3 + K$$

$$\Rightarrow I = -\frac{2}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\log\left(1+\frac{1}{x^2}\right) + \frac{2}{9}\left(1+\frac{1}{x^2}\right)^{3/2} + K$$

2.

$$I = \int \cos(\log x)\,dx$$

$$\Rightarrow I = \int x \cdot \frac{\cos(\log x)}{x}\,dx$$

Put $\log x = t$

$\Rightarrow \frac{dx}{x} = dt$

and $x = e^t$

$$= \int e^t \cos t\,dt$$

$$= \cos t\int e^t dt - \int\left[\frac{d}{dx}(\cos t)\int e^t dt\right]dt$$

$$= e^t\cos t + \int e^t \sin t\,dt$$

$$= e^t\cos t + \sin t\int e^t dt - \int\left[\frac{d}{dt}(\sin t)\int e^t dt\right]dt$$

$$= e^t\cos t + \sin t\, e^t - \int e^t\cos t\,dt$$

$$\Rightarrow I = e^t(\cos t + \sin t) - I$$

$$\Rightarrow 2I = e^t(\cos t + \sin t) + k$$

$$\Rightarrow I = \frac{1}{2}e^t(\cos t + \sin t) + K_2$$

3. $I = \int \frac{2x^{12} + 5x^{9}}{(x^5 + x^3 + 1)^3} dx$

$\Rightarrow I = \int \frac{2x^{12} + 5x^{9}}{x^{15}\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3} dx$

$= \int \left(2\frac{x^{12}}{x^{15}} + \frac{5x^{9}}{x^{15}}\right) \frac{dx}{(1 + x^{-2} + x^{-5})^3}$

$= \int \frac{2x^{-3} + 5x^{-6}}{(1 + x^{-2} + x^{-5})^3} dx$

Put $1 + x^{-2} + x^{-5} = z$

$\Rightarrow \left(-\frac{2}{x^3} - \frac{5}{x^6}\right) dx = dz$

$\Rightarrow (2x^{-3} + 5x^{-6})\, dx = -dz$

$\Rightarrow I = \int \frac{-dz}{z^3} = (-)\frac{z^{-3+1}}{-3+1} + k$

$= \frac{1}{2z^2} + k = \frac{1}{2(1 + x^{-2} + x^{-5})^2} + k$

4.

$$I = \int \frac{\sin^2 x \cos^2 x \, dx}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2}$$

$$\Rightarrow I = \int \frac{\sin^2 x \cos^2 x \, dx}{[\sin^3 x(\sin^2 x + \cos^2 x) + \cos^3 x(\sin^2 x + \cos^2 x)]^2}$$

$$= \int \frac{\sin^2 x \cos^2 x}{(\sin^3 x + \cos^3 x)^2} \, dx$$

$$= \int \frac{\frac{\sin^2 x \cos^2 x}{\cos^6 x}}{\left(\frac{\sin^3 x + \cos^3 x}{\cos^3 x}\right)^2} \, dx$$

$$= \int \frac{\tan^2 x \sec^2 x \, dx}{(1 + \tan^3 x)^2}$$

Put $1 + \tan^3 x = z$

$\Rightarrow 3 \tan^2 x \sec^2 x \, dx = dz$

$$\Rightarrow I = \frac{1}{3} \int \frac{dz}{z^2} = -\frac{1}{3z} + K$$

$$= K - \frac{1}{3}(1 + \tan^3 x)^{-1}$$